Probability distributions of travel times on arterial networks: a traffic flow and horizontal queuing theory approach

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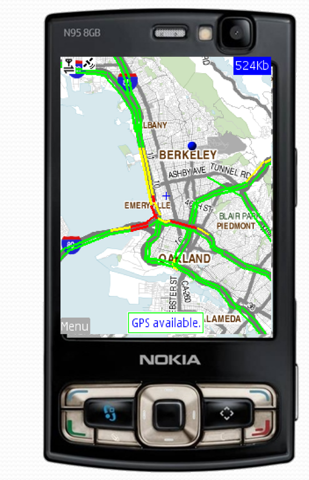
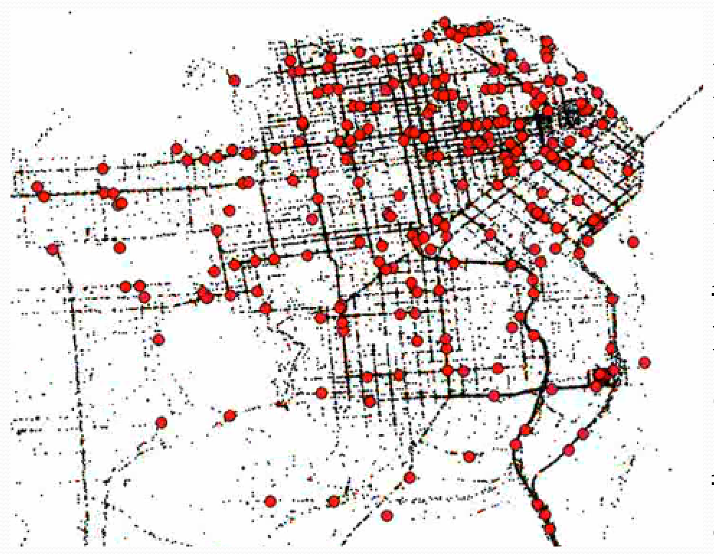


1) Data: sparse measurements from probe vehicles

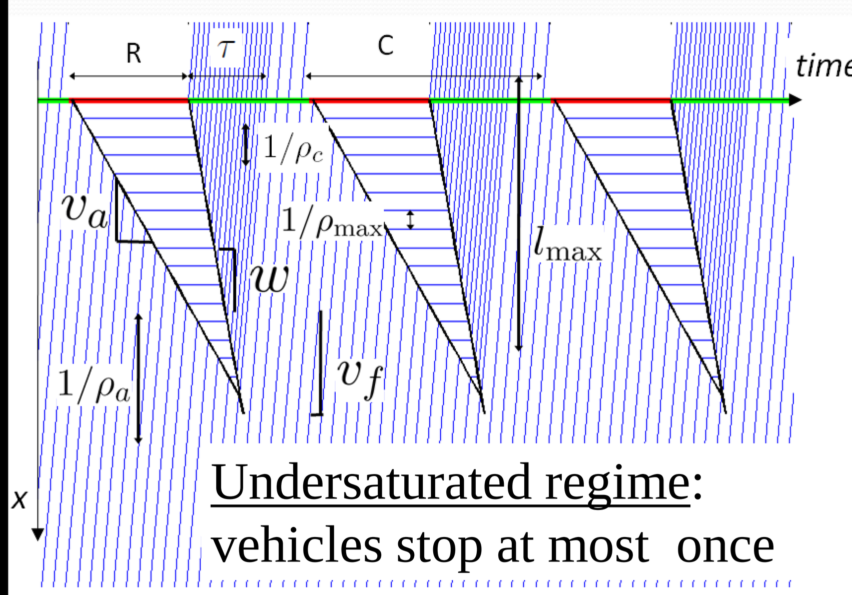


Data collected from a fleet of **500 vehicles**, reporting their location every minute (San Francisco, CA).
Black dots: Cumulative measurements received between 12am and 7am, on March 29th, 2010.
Red circles: Location of the probe vehicles at 7am on that day.

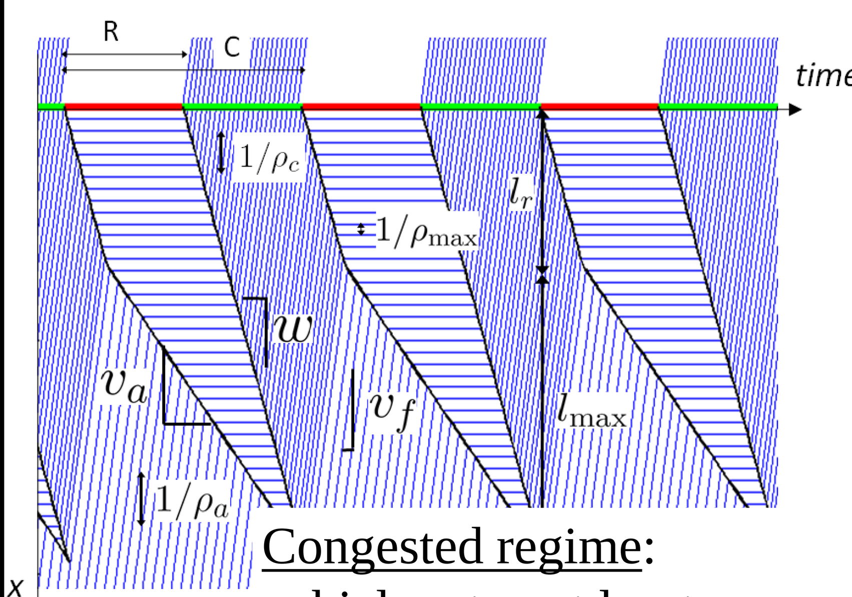
Challenges of arterial traffic estimation from sparse probe measurements:
- Dynamics driven by the presence of ***signals*** with ***unknown*** parameters
- Trade-off between the amount of data available in real-time and the information that can be reconstructed
- Variations in the travel time under similar conditions:
  - The delay depends on the entrance time on the link
  - Differences in driving behavior influence the travel time

Our approach:
➔ Parametric probability distribution of travel times which represent the dynamics over several cycles.
➔ The parameters represent traffic characteristics which are learned from historical and real-time data.

2) Dynamics: horizontal queuing model



Undersaturated regime: vehicles stop at most once

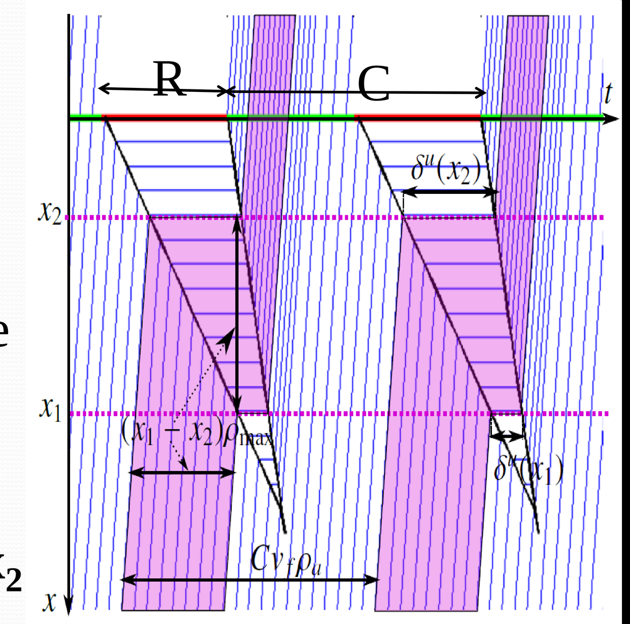


Congested regime: vehicles stop at least once

Model:
- Conservation of the vehicle and triangular fundamental diagram
- Uniform arrivals
- Periodic dynamics (period is duration of cycle: **C**)

Parameters:
- *Traffic signal*: red time R and cycle time C,
- *Driving behavior*: free flow pace pf with distribution p
- *Saturation queue length*: distance traveled between successive stops in the congested regime
- *Queue length*: distance to the downstream intersection of the last vehicle which stops in the queue.

3) Delay: Analytical derivations



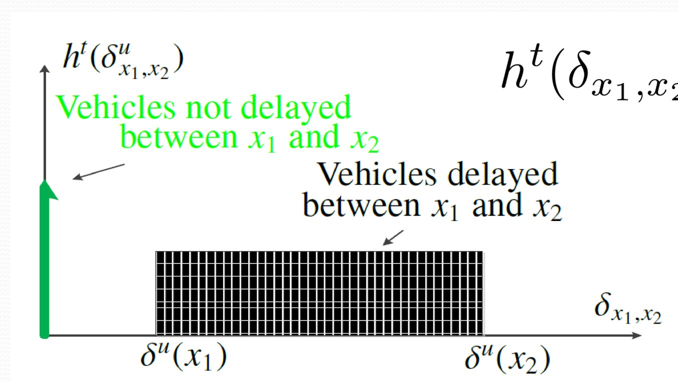
Delay at location x:
- Maximum at x=0 (red time), null at the end of the queue.
- Decreases linearly with the position x in the queue

$$\delta^u(x) = R\left(1 - \frac{\min(x, l_{\max})}{l_{\max}}\right)$$

Proportion of stopping vehicles between x_1 and x_2 (among the vehicles entering in a cycle)

$$\eta^u_{x_1,x_2} = \frac{\min(x_1, l_{\max}) - \min(x_2, l_{\max})}{l_{\max}}\left(\frac{R}{C} + \left(1 - \frac{R}{C}\right)\frac{l_{\max}}{l^s_{\max}}\right)$$

Probability distribution of delay:



$$h^t(\delta_{x_1,x_2}) = \boxed{(1-\eta^u_{x_1,x_2})\mathrm{Dir}_{\{0\}}(\delta_{x_1,x_2})} + \boxed{\frac{\eta^u_{x_1,x_2}}{\delta^u(x_2)-\delta^u(x_1)}\mathbf{1}_{[\delta^u(x_1),\delta^u(x_2)]}(\delta_{x_1,x_2})}$$

In the congested regime: different cases depending on the relative location of x_1 and x_2 with respect to the queue length and remaining queue length

4) Probability distribution of travel times

Travel time = Delay + Free flow travel time
- Free flow travel time: proportional to the distance traveled and free flow pace
Scaling of the pdf of free flow pace

$$p_f \sim \varphi^p(p_f) \Rightarrow \varphi^y_{x_1,x_2}(y_{f;x_1,x_2}) = \varphi^p\left(\frac{y_{f;x_1,x_2}}{x_1-x_2}\right)\frac{1}{x_1-x_2}$$

- Sum of independent random variables
➔ Convolution product

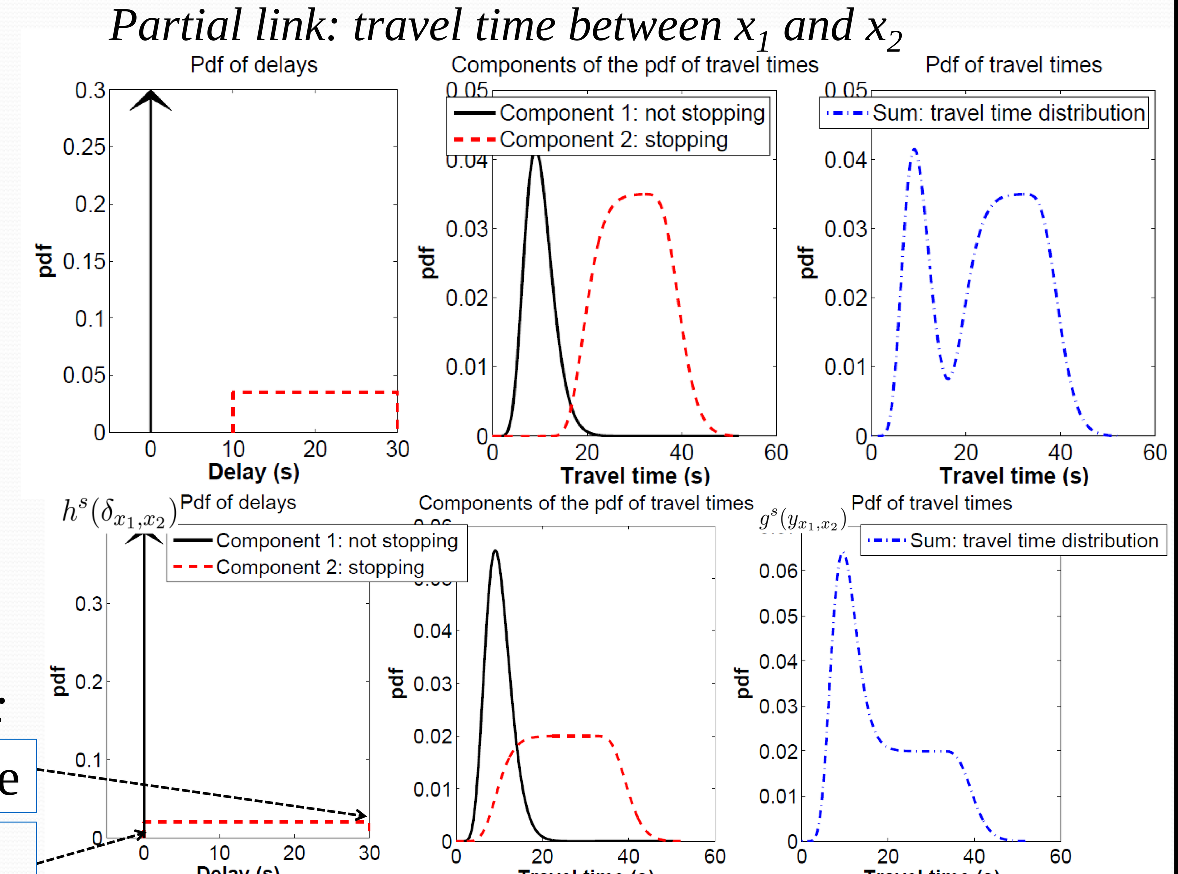
- Linearity of the convolution
➔ Compute the pdf of travel times for each type of delay (e.g. stopping vs. non stopping vehicles)

Partial link: travel time between x_1 and x_2

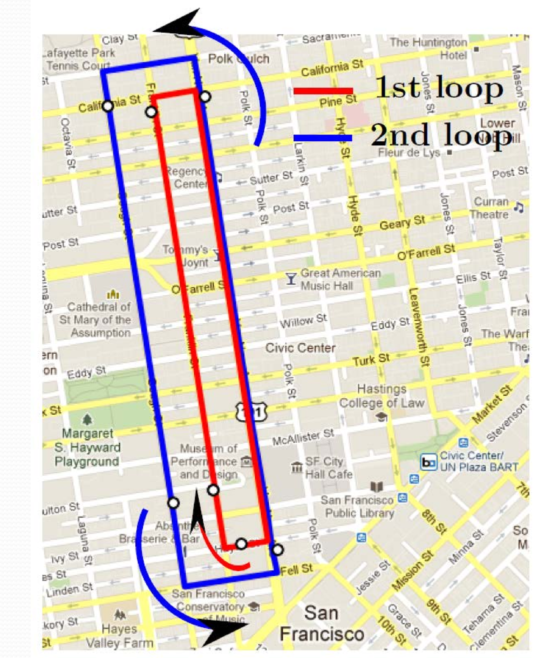
Entire link:

Max. delay: Red time

Min. delay: Zero

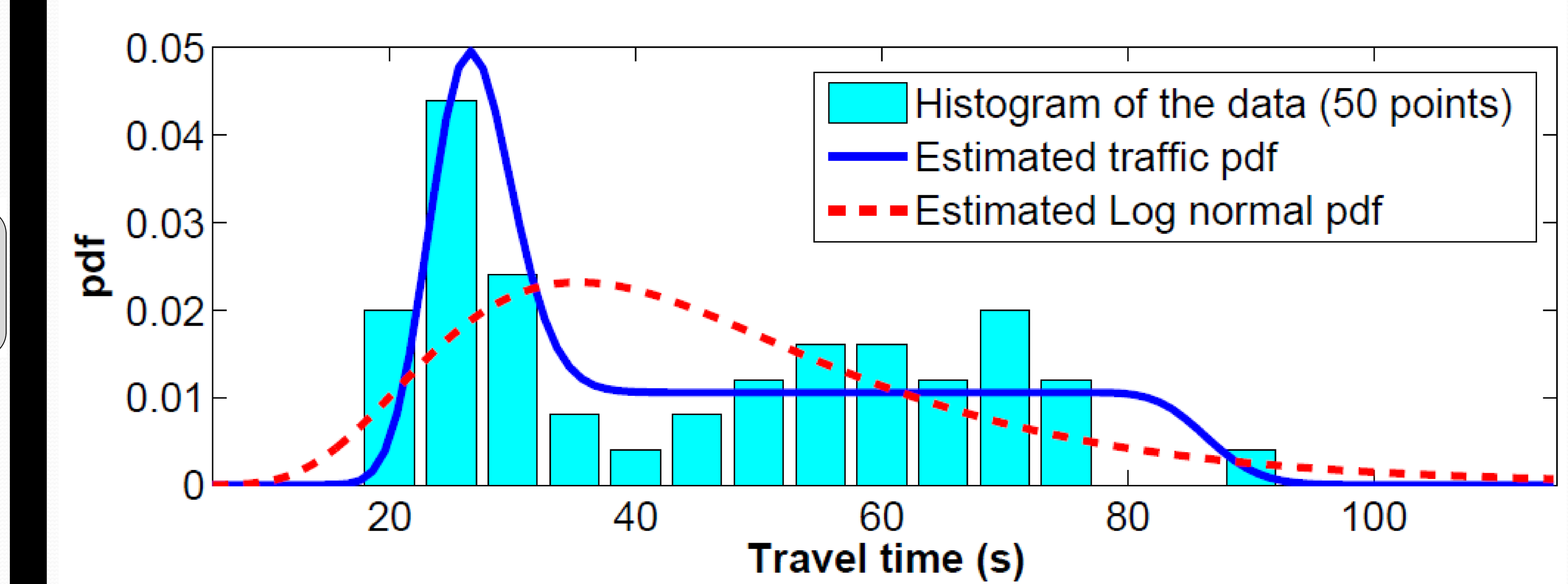


5) Numerical experiment



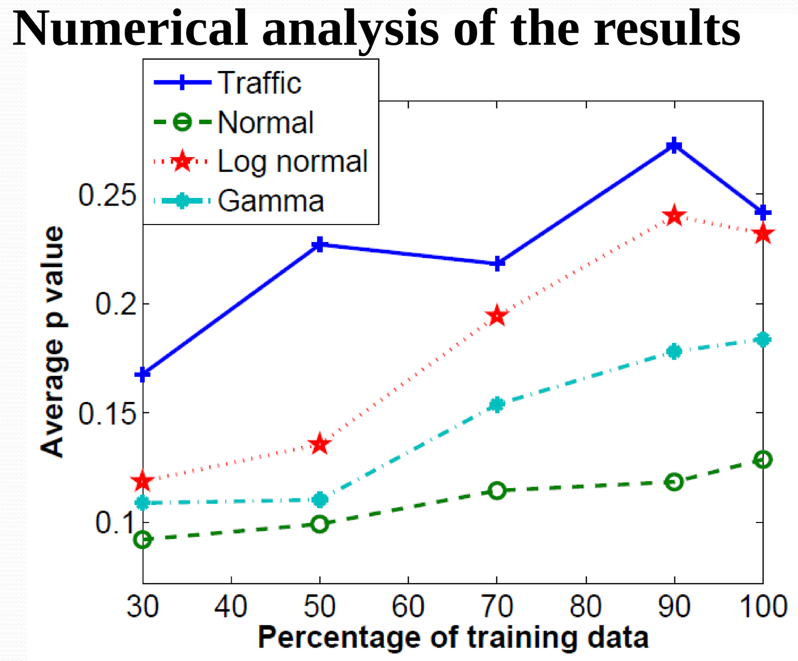
Field test experiment (San Francisco, CA):
- 20 drivers, 3 hours of test on 3 consecutive days
- 2 distinct loops (1.9 and 2.3 miles)
- Trajectory measurements (GPS device)

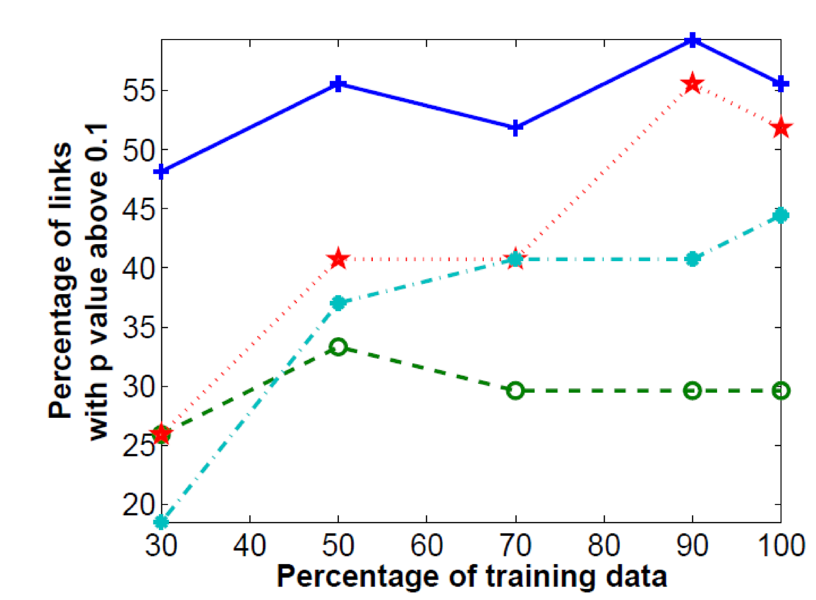
Numerical analysis:
- Down-sampling of trajectories to simulate sparse probe measurements
- Learning on the training set: *maximum likelihood* estimation of the distribution parameters
- Validation on the remaining data



6) Analysis of the results and current directions

Numerical analysis of the results





Test the *hypothesis* that travel times are distributed according to different classes of distributions. The ***traffic distribution*** fits the validation data best, in particular when little data is available.

Current directions:
Model the conservation of vehicle at junction to derive a model of the stochastic evolution of queue length (i.e. congestion) across the network.