A Three-Stream Model for Arterial Traffic

C. Bails, A. Hofleitner, Y. Xuan and A. Bayen

1) Introduction and motivation

- Traffic flow model for traffic dynamics at signalized intersections
- Dynamics driven by the formation and dissolution of queues at traffic signals
- Variations in the travel time under similar conditions:
  - ➢ The delay depends on the entrance time on the link

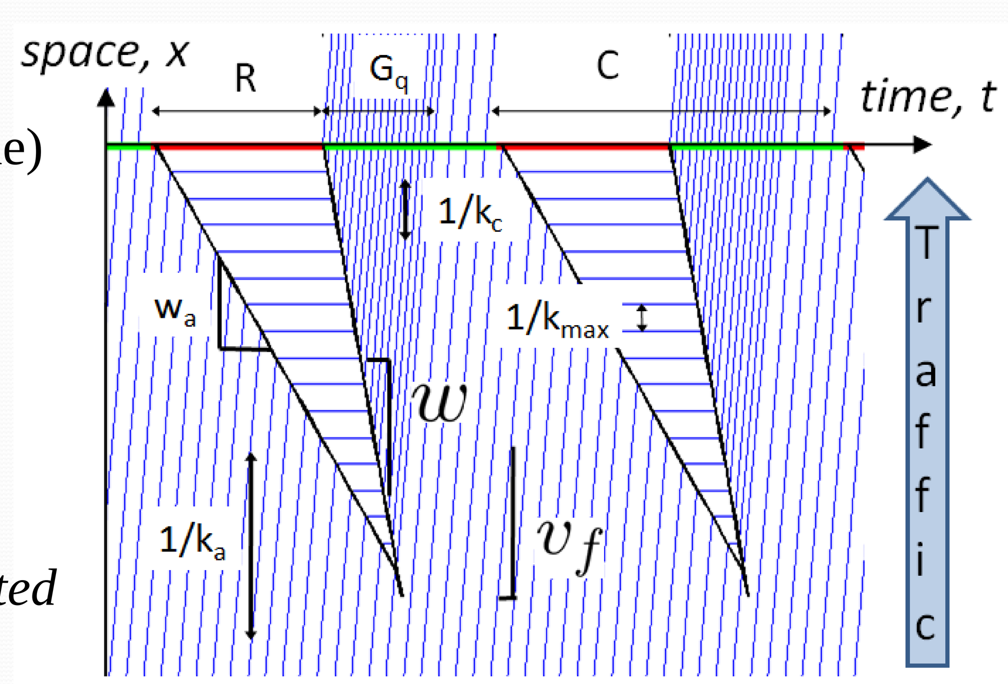
Our approach:

➔ Lighthill-Whitham-Richards model with triangular fundamental diagram.
➔ Analytical model with limited number of parameters
➔ The parameters represent traffic characteristics, which can be learned, measured or estimated.
➔ Analytical derivation of the total delay depending on the parameters.
➔ Traffic signal optimization on arterial corridors (analytical derivation of the optimal control)

Stream of vehicles (k, T):
Group of vehicles characterized by a uniform density k. The duration T of the stream is the time it takes for all vehicles within the stream to go through a point in space.

2) Three-stream model for arterial traffic

Constant arrival flow = three streams departure flow:
- Null flow (red time)
- Capacity (queue dissipation)
- Arrival flow (extra green time)

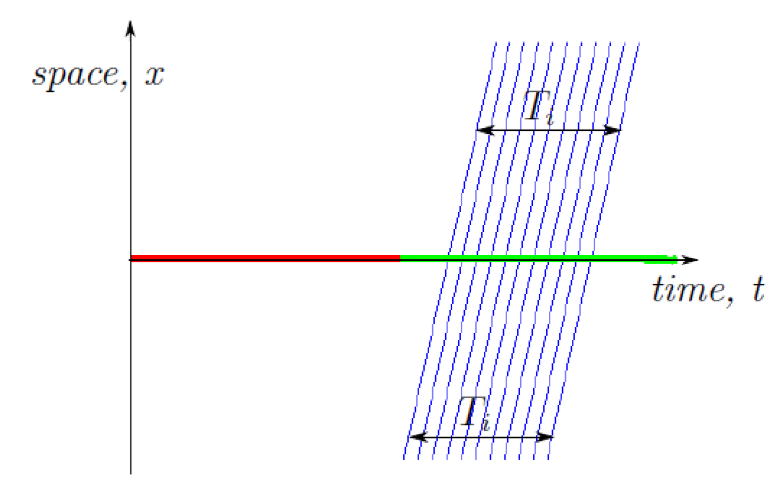


General arrival flow ≈ three streams departure flow:
- Null flow (red time)
- Capacity (queue dissipation)
- *Average* arrival flow (extra green time), *approximated* as a platoon. Density of the platoon computed from the conservation of vehicles

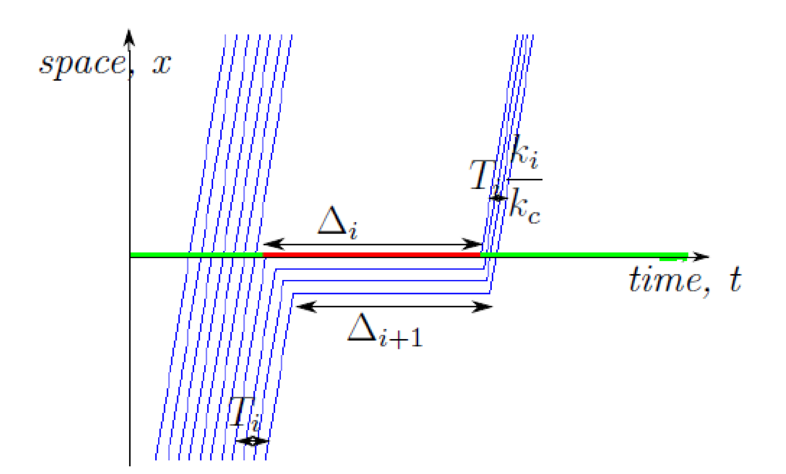
Density of platoon during residual green time computed from the conservation of vehicles:

$$\underbrace{\sum_{i=1}^{3} v_f k_i T_i}_{\text{Arrival streams}} = \underbrace{0 \cdot R}_{\text{Red time}} + \underbrace{v_f k_c G_q}_{\text{Queue dissipation time}} + \underbrace{v_f k_f (C-R-G_q)}_{\text{Residual green time}}.$$

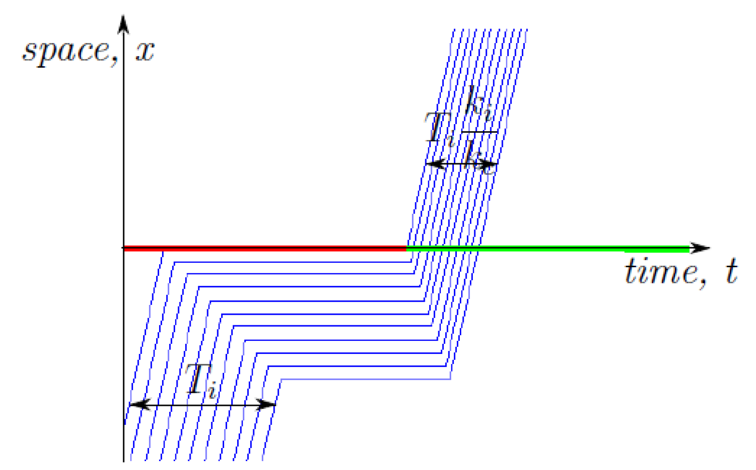
3) Dynamics of a stream through an intersection



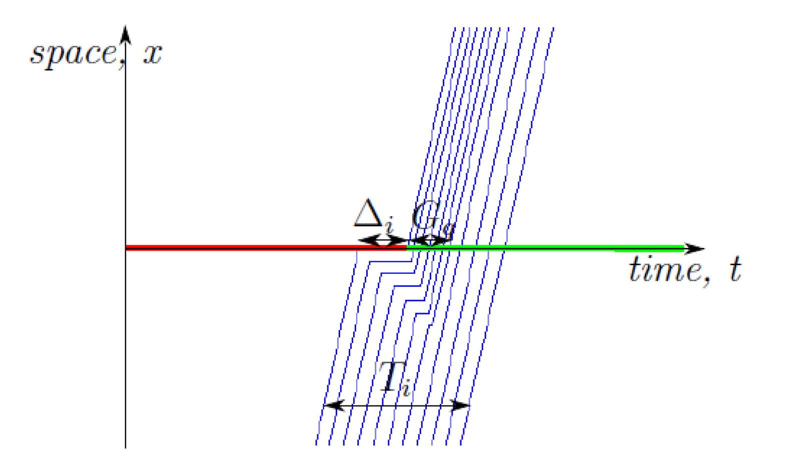
No vehicle stops at the signal



The first vehicles (fraction α of the stream) do not stop.

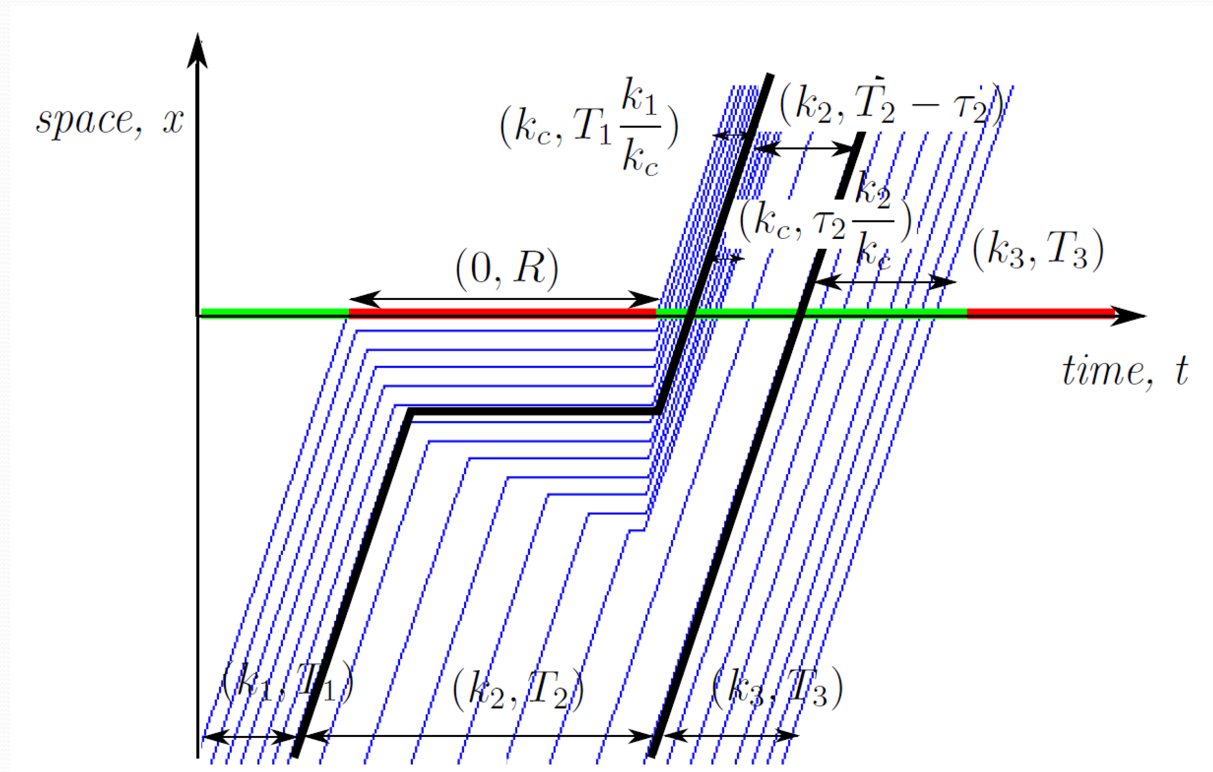


All vehicles stop at the signal



The last vehicles (fraction α of the stream) do not stop.

4) Input/output relation in a three stream model



$$\overbrace{\begin{Bmatrix} (k_1, T_1) \\ (k_2, T_2) \\ (k_3, T_3) \end{Bmatrix}}^{\text{Arrival streams}} \longmapsto \overbrace{\begin{Bmatrix} (0, R) \\ (k_c, G_q) \\ (k_f, C-R-G_q) \end{Bmatrix}}^{\text{Averaged departure streams}}$$

5) Optimal traffic signal coordination

Optimization of the offset to minimize the total delay experienced by the vehicles:

$$D = \int_0^C W(t)q(t)dt = \int_0^C W(t)v_f k(t)dt$$

Objective function is quasi-convex with respect to the decision variable
➔Unique solution computed analytically

Optimization on an arterial corridor:
After passing through a few intersections, the output of the three-stream model converges to a stationary optimal control corresponding to a green wave coordination.

Validation using CORSIM (micro-simulation):
- Analytical computation of the optimal control
- Bounded error between three stream model approximation and micro-simulation results
- Better understanding of traffic dynamics

6) Conclusion and discussions

Analytical model of arterial traffic with limited number of parameters (growing linearly with the number of links)

Traffic signal optimization:
- Analytical solution to the classical problem of traffic light optimization
- Analytical optimal control strategies for the choice of the offsets, as a function of the arrival streams
- Real-time implementation with timely adaptation of the control strategies as traffic conditions change (all computations done off-line)

Arterial traffic modeling:
- Better understanding of traffic dynamics
- Characterization of travel time distributions both on links and route in a tractable fashion.
- Improvement compared to current model which assume constant arrival rates and do not take into account signal synchronization.